HEAT TRANSFER

Unit VI

Radiation Heat Transfer

Radiation heat transfer is defined as, “ **the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused solely by a temperature difference.**

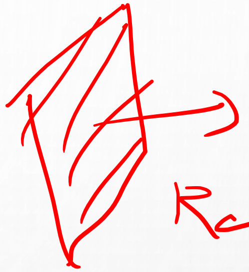
Conduction and convection requires medium but radiation does not require medium.

Rate of heat transfer by conduction and convection varies as temperature difference to the first power whereas radiant heat exchange between two bodies varies as temperature difference to the fourth power.

The emission of the thermal radiation depends upon the nature, temperature and state of the emitting surface.

Total emissive power:-

- It is defined as total amount of radiation emitted by a body per unit area and time.



E_b Radiated $\frac{W}{m^2}$

Monochromatic emissive power:-

- it is defined as the rate of energy radiated per unit area of the surface per unit wavelength.

$E_{b\lambda}$

Emissivity :- (ε)

- It is defined as the ratio of the emissive power of the any body to the emissive power of the black body.

$\varepsilon = \frac{E}{E_b}$

$0 < \varepsilon < 1$

Transparent Reflective Absorptive

Absorptivity, Reflectivity, and Transmissivity

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that every body, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths. Recall that radiation flux *incident on a surface* is called **irradiation** and is denoted by G.

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Figure 11–31. *The fraction of irradiation absorbed by the surface* is called the **absorptivity** α, *the fraction reflected by the surface* is called the **reflectivity** ρ, and *the fraction transmitted* is called the **transmissivity** τ. That is,

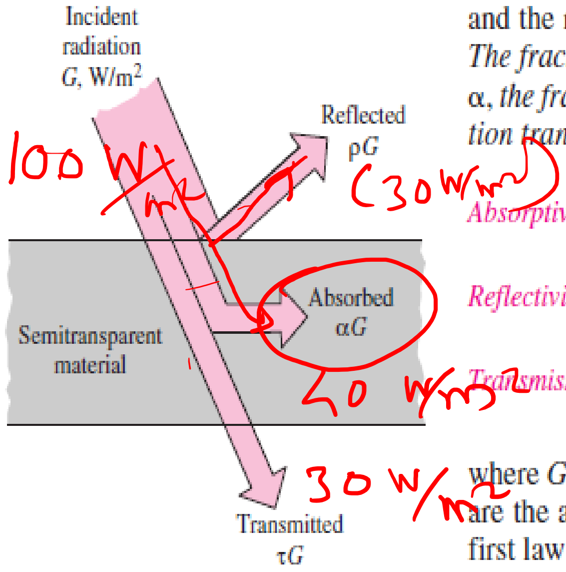


FIGURE 11–31
The absorption, reflection, and transmission of incident radiation by a semitransparent material.

Absorptivity: $$\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{abs}}{G}, \quad 0 \le \alpha \le 1 \quad \text{(11-37)}$$

Reflectivity: $$\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{ref}}{G}, \quad 0 \le \rho \le 1 \quad \text{(11-38)}$$

Transmissivity: $$\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{tr}}{G}, \quad 0 \le \tau \le 1 \quad \text{(11-39)}$$

where G is the radiation energy incident on the surface, and G_{abs}, G_{ref}, and G_{tr} are the absorbed, reflected, and transmitted portions of it, respectively. The first law of thermodynamics requires that the sum of the absorbed, reflected, and transmitted radiation energy be equal to the incident radiation. That is,

$$G_{abs} + G_{ref} + G_{tr} = G \quad \text{(11-40)}$$

Dividing each term of this relation by G yields

$$\alpha + \rho + \tau = 1 \quad \text{(11-41)}$$

30 + 30 + 40 = 100
ref. tran abs

$\alpha = \frac{40}{100} = 0.4$

$\rho = \frac{30}{100} = 0.3$

$\tau = \frac{30}{100} = 0.3$

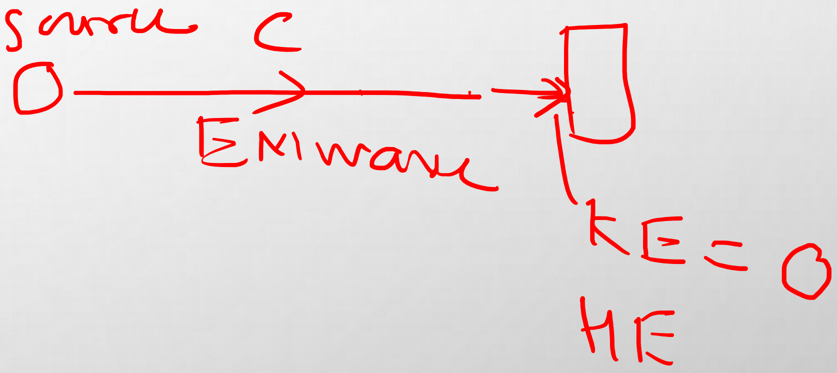
$\alpha + \rho + \tau = 1$

$0.4 + 0.3 + 0.3 = 1$

Black body: For perfectly absorbing body,	$\alpha = 1$, $\tau = 0$, $\rho = 0$, such a body is called a black body, which neither reflects nor transmits any part of the incident radiation but absorbs all of it. In practice perfectly black body does not exist.
Opaque Body: For opaque body,	$\tau = 0$, $\alpha + \rho = 1$, such a body is called opaque body which does not transmits but only absorbs or reflects the incident radiation.
White body: For white body, (Mirror)	$\rho = 1$, $\alpha = 0$, $\tau = 0$, such a body is called white body which reflects all radiation falling on it.
Grey body: if the radiative properties ρ, α, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called grey body.	A grey body is also defined as one whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation.

- Every surface emits electromagnetic waves continuously in all possible directions due to its temp. (it is above abs. zero temp)

- These electromagnetic waves carry energy when they propagate and transfer thermal energy when they impinge on a substance/ body. This kind of energy transfer is called Heat Transfer by RADIATION.



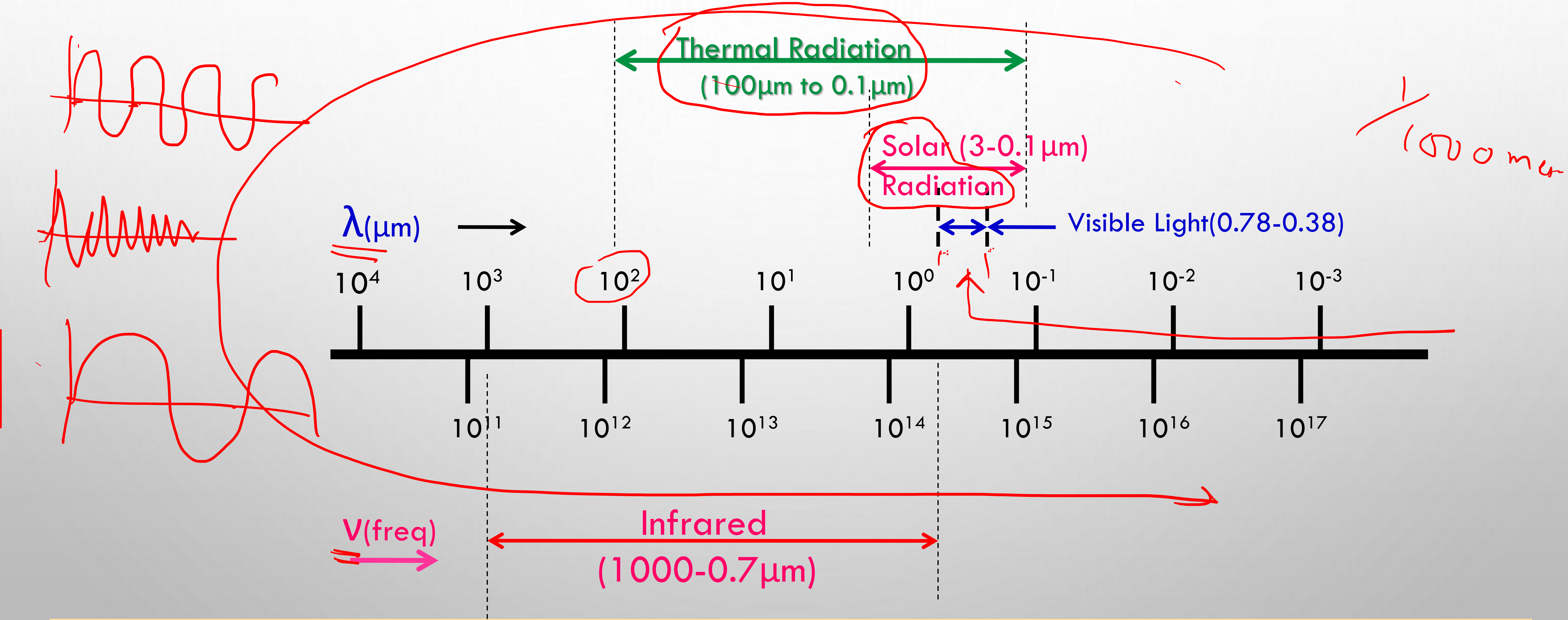
- Heat transfer by emissions of radiation is explained by two theories:

Wave Theory or Maxwell's Classical Theory

- Radiation emissions propagate in the form of waves. Since waves propagate through some medium, this theory assumes that Universe is filled with a hypothetical medium ETHER.
- Waves travel with the speed of light
- Every wave possesses certain amount of energy, a part of which is transferred on being impinged by some object in its route of travel

1 wave —transmission→ 50% 50% impingement

Spectrum of Electromagnetic Radiation



2.Quantum Theory or Planck's Theory

- Radiation emissions are in the form of series of entities known as quanta.
- Each quanta possesses certain amount of energy, which is proportional to its frequency of emission.
- Quanta moves with the speed of light and releases its energy on being impinged by some object in its route of travel

Properties of Surface

Reflectivity (ρ):

Fraction of total energy falling on the surface, which is reflected

Absoptivity (α):

Fraction of total energy falling on the surface, which is absorbed

Transmissivity (τ):

Fraction of total energy falling on the surface, which is transmitted (through)

$$Hence, \rho + \alpha + \tau = 1$$

Some Definitions

Black Body:

A body which absorbs all incident energy and does not transmit and reflects at all, is called Black Body. It is also the highest emitter of radiation

$$\tau = 0; \rho = 0; \alpha = 1; \varepsilon = 1$$

Examples: Surface coated with lamp black, milk, ice, water, white paper etc

White Body:

A body which reflects the entire radiation falling on it, is called White Body

$$\tau = 0; \alpha = 0; \varepsilon = 0; \rho = 1$$

Gray Body:

The body having same value of emissivity at all wavelengths , which is equal to average emissivity, is known as Grey body.

Generally, all engg metals are grey bodies, for which $\varepsilon=\alpha$, when in thermal equilibrium

Emissive Power (q):

It is the rate, at which the radiant flux is emitted from the surface at certain temp

Monochromatic Emissive Power (q_λ):

It is the rate, at which radiant flux is emitted with a specific wavelength at certain temp; it is λ dependent emissive power

Emissivity (ε):

It is the ratio of emissive power of a surfaceto that of black body when both at same temp

$$\varepsilon = \frac{q}{q_b}$$

Monochromatic Emissivity (ε_λ):

It is the ratio of monochromatic emissive power of a surface to that of black body when both are at same temp for same given wavelength

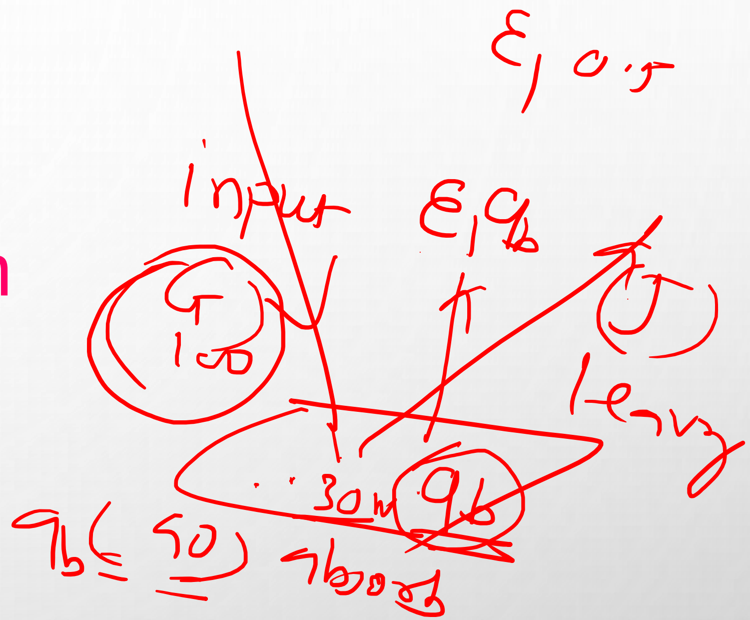
$$\varepsilon_\lambda = \frac{q_\lambda}{q_{b_\lambda}}$$

(Non black) / (Black)

Radiosity (J):

It is the net energy leaving the surface.
It consists of the radiant energy emitted and energy reflected out of the incident radiation from the surface.

$$J = \varepsilon_1 q_b + (1-\varepsilon_1)G$$



Irradiation (G):

It is the net energy incident/falling on the surface (need not necessarily be absorbed)

Planck's Law

Planck's law is based on Quantum theory and it gives the relationship among monochromatic Emissive power of black body, the absolute Temp of the surface and corresponding Wavelength and is given as:

$$q_{b\lambda} = \frac{2\pi C_1}{\lambda^5 . \left(e^{C_2/\lambda T} - 1\right)} W/m^2;$$

$$\text{where} C_1 = 0.596x10^{-16} \& \ C_2 = 0.014387$$

Planck's Law

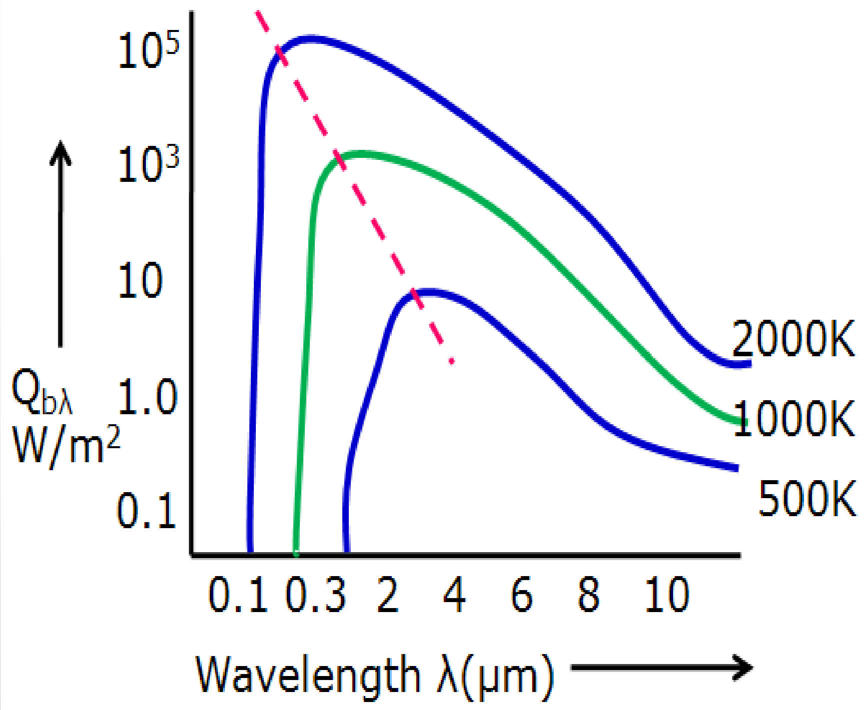
Plot shows the following:

- $q_{b\lambda}$ at certain temp first increases with λ, attains some max value and then decreases

- For specific wavelength, $q_{b\lambda}$ of black surface increases with temp

- Most of the thermal radiations lie in wavelength region from 0.3 to 10 μm

- Wavelength (λ_m) , at which peak $q_{b\lambda}$ obtained, decreases with increase in temp

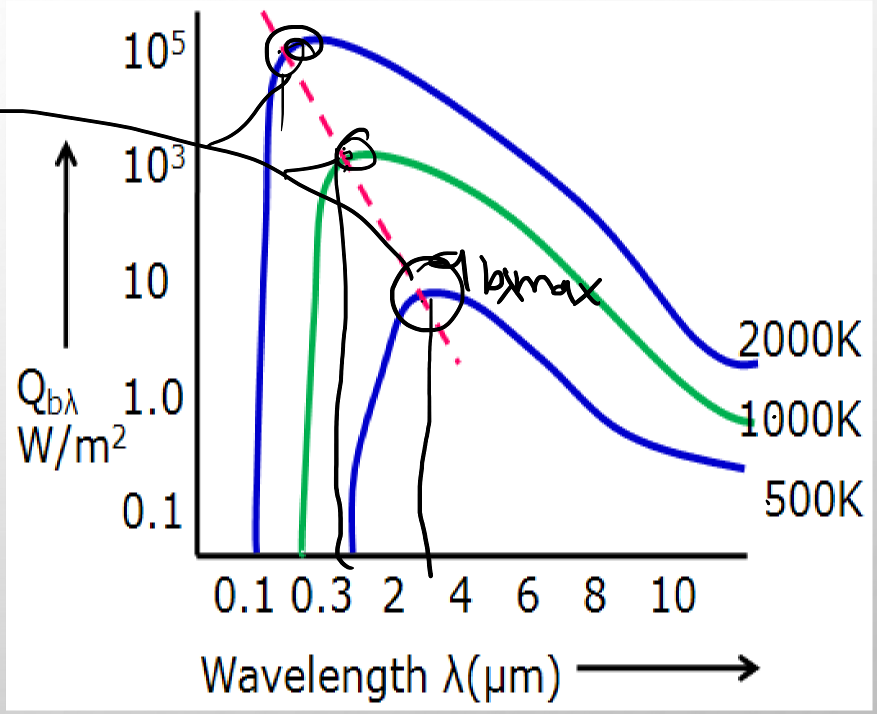


Wien's Displacement Law

Wien's Law gives the relationship between the wavelengths (λ_m) , at which peak ($q_{b\lambda}$) monochromatic emissive power is obtained and the absolute temp and given as:

$$\lambda_m T = 0.0029 \text{ mK}$$

$T\uparrow \quad \lambda_m \downarrow$



Plot and above relation show that the value of wavelength, at which peak/max monochromatic emissive power is obtained, decreases (displaces/shifts) with increase in surface temperature of the black body.

Derivation of Wien's Law

As per Planck's law, $q_{b\lambda} = \dfrac{2\pi C_1}{\lambda^5\left(e^{C_2/\lambda T} - 1\right)}$

$$Putting\ \frac{C_2}{\lambda T} = x \Rightarrow \lambda = \frac{C_2}{xT}$$

$$Substituting\, q_{b\lambda} = \frac{2\pi C_1}{\dfrac{{C_2}^5}{x^5 T^5}(e^x - 1)}$$

$$Or\ q_{b\lambda} = \frac{2\pi C_1 . x^5 . T^5 . (e^x - 1)^{-1}}{{C_2}^5}$$

This eqn expresses $q_{b\lambda}$ of black body as a function of x

Derivation of Wien's Law

For obtaining the wavelength (λ_m) for specified temp, at which max $q_{b\lambda}$ occurs, we have to differentiate this equation wrt x and equate it to zero.

$$\therefore \frac{d}{dx}\left[\frac{2\pi C_1 . x^5 . T^5 . (e^x - 1)^{-1}}{{C_2}^5}\right] = 0$$

$$Or \frac{2\pi C_1 T^5}{{C_2}^5} . \frac{d}{dx}[x^5 . (e^x - 1)^{-1}] = 0$$

$$Or \frac{d}{dx}[x^5 (e^x - 1)^{-1}] = 0$$

Derivation of Wien's Law

$$\frac{d}{dx}[x^5(e^x-1)^{-1}]=0$$

$$\Rightarrow (e^x-1)^{-1}.(5x^4)+(x^5).(-1).(e^x-1)^{-2}.e^x=0$$

$$\Rightarrow \frac{5x^4}{(e^x-1)}-\frac{x^5.e^x}{(e^x-1)^2}=0$$

$$\Rightarrow \frac{x^4}{(e^x-1)}\left[5-\frac{x.e^x}{(e^x-1)}\right]=0$$

$$Or\ \frac{5e^x-5-x.e^x}{e^x-1}=0 \Rightarrow e^x(5-x)-5=0$$

Derivation of Wien's Law

$$Wenowhave \rightleftarrows e^x(5-x)-5=0$$

This eqn is satisfied by putting x=4.96

$$Hence, x = 4.96 = \frac{C_2}{\lambda T}$$

$$\therefore \lambda_m T = \frac{0.014387}{4.96} = 0.0029$$

$$Therefore, \lambda_m T = 0.0029 mK$$

Stefan Boltzmann's Law

Emissive power of a black body is directly proportional to fourth power of its absolute temperature:

$$q_b \infty T^4 or q_b = \sigma T^4;$$
$$where \sigma = 5.67x10^{-8} W/m^2K^4$$

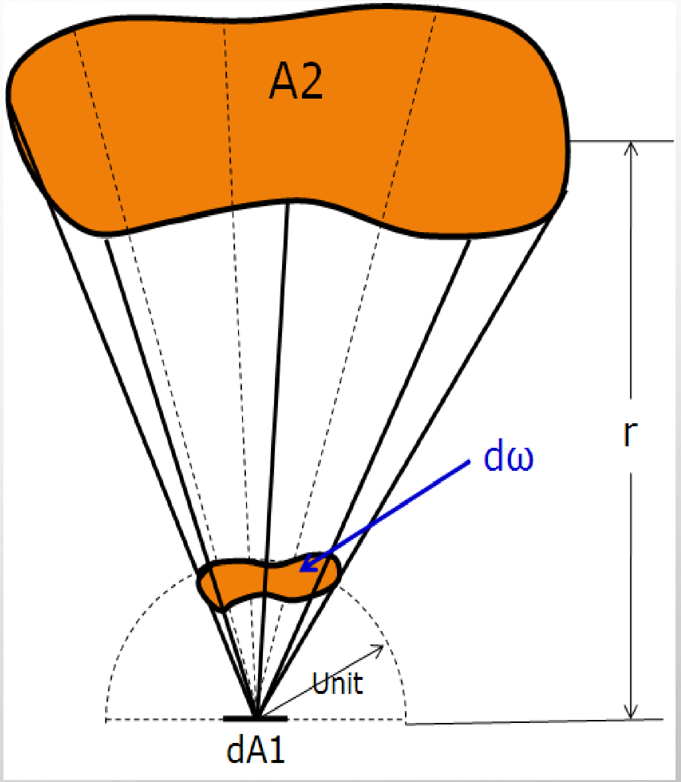
Kirchhof's Law

When a surface is in thermal equilibrium with its surroundings, the emissivity of the surface is equal to its absorptivity.

$$That\ is\ \alpha = \varepsilon$$

Solid Angle

Solid angle subtended by surface A_2 at surface dA_1 (elementary surface) is numerically equal to the area on a surface of sphere with unit radius and centre at elementary area, which is cut by conical surface having its base as perimeter of A_2 and vertex at dA_1



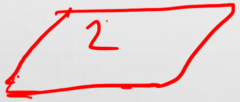
Solid angle is measured in Steradians (Sr) and denoted by symbol ω

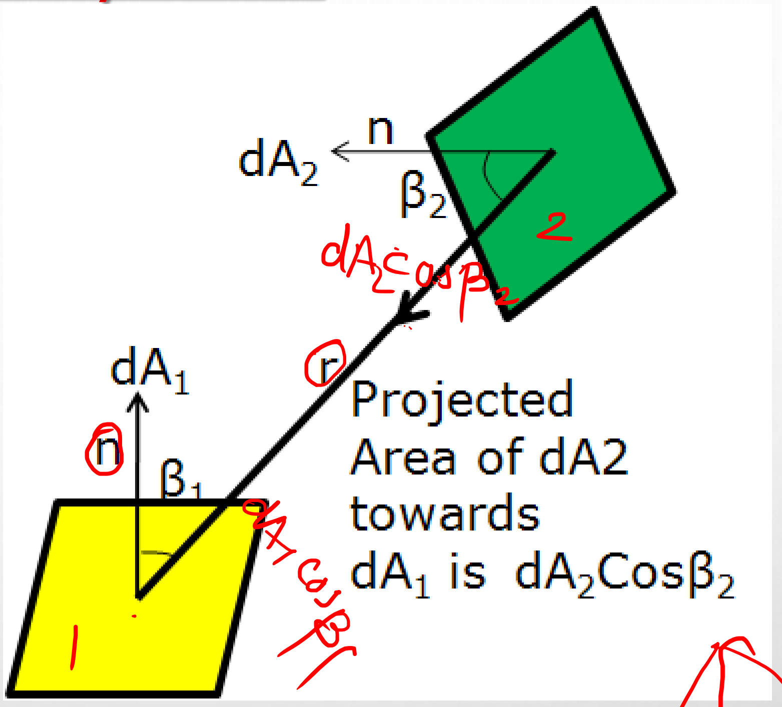
$$\therefore d\omega = \frac{A_2}{r^2}$$

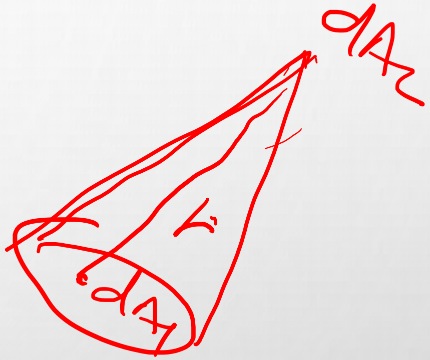
Solid Angle Between Two Elementary Areas

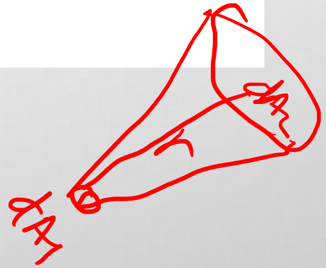
Solid Angle subtended by elementary area dA_2 at dA_1 can be given as:

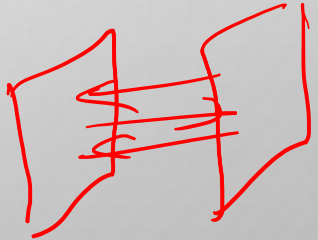
$$d\omega = \frac{dA_2 Cos\beta_2}{r^2}$$









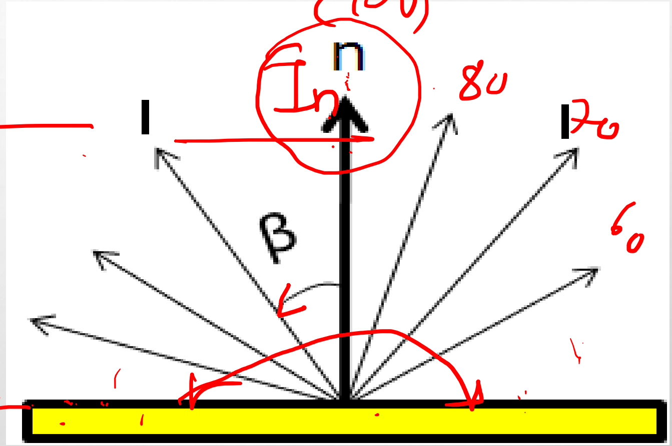


Similarly, solid angle subtended by area dA_1 at dA_2 can be given as:

$$d\omega = \frac{dA_1 Cos\beta_1}{r^2}$$

Intensity of Radiation

- Intensity of radiation emitted by a surface is equal to the radiant energy passing in a specified direction per unit solid angle
- Intensity of radiation varies in different directions and is max in the direction normal to the surface



Lambert Cosine Law:

Intensity of radiation in any direction is proportional to the Cosine of the angle made by that direction with the normal.

That is, $I = I_n \cos\beta$; where I_n is the intensity (max) in the normal direction and β is the angle made by that direction with the normal

$$Total\ emissive\ power\ q = \pi I_n \Rightarrow I_n = \frac{\sigma T^4}{\pi}$$

Shape Factor/Geometric Factor

Shape factor is defined as the fraction of energy emitted by one surface and directly intercepted by the other.

$$ShapeFactor: F_{12} = \frac{1}{A_1}\left[\int_{A_1}\int_{A_2} \frac{Cos\beta_1 Cos\beta_2 dA_1 dA_2}{\pi r^2}\right]$$

Shape Factor depends upon:

1. Shape and size of surfaces
2. Orientation of surfaces wrt each other
3. Distance between the surfaces

Relations/ Theorems of Shape Factors

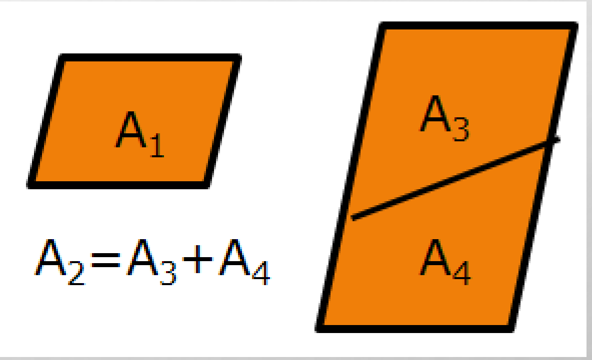
1. **Reciprocal Relation:** $F_{12}.A_1 = F_{21}.A_2$

2. **Enclosure Relation:** If n no of surfaces form an enclosure, then:

$$F_{11}+F_{12}+F_{13}+\ldots\ldots\ldots+F_{1n}=1$$
$$F_{21}+F_{22}+F_{23}+\ldots\ldots\ldots+F_{2n}=1$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$F_{n1}+F_{n2}+F_{n3}+\ldots\ldots\ldots+F_{nn}=1$$

3. **Summation Relation:**

Shape Factor F_{12} between two surfaces A_1 and A_2 is equal to the sum of shape factors F_{13} & F_{14}, if the two areas A_3 & A_4 together make up area A_2



$$\therefore F_{12} = F_{13} + F_{14}; However, F_{21} \neq F_{31} + F_{41}$$

Relations/ Theorems of Shape Factors

4. Shape factor depends on geometry and orientation of surfaces and it does not change with temp.
5. Shape Factor wrt itself (F_{11}, F_{22}, F_{33}…) means radiation emitted by a portion of a surface falling on the other portion of itself directly

   Example : Shape Factor for concave surface

   Shape factor for convex or Flat surface wrt itself is zero.

Radiation Heat Exchange Between Two Parallel Plates

Consider two grey opaque parallel plates maintained at temperatures T_1 & T_2 having emissivities ε_1 & ε_2 respectively

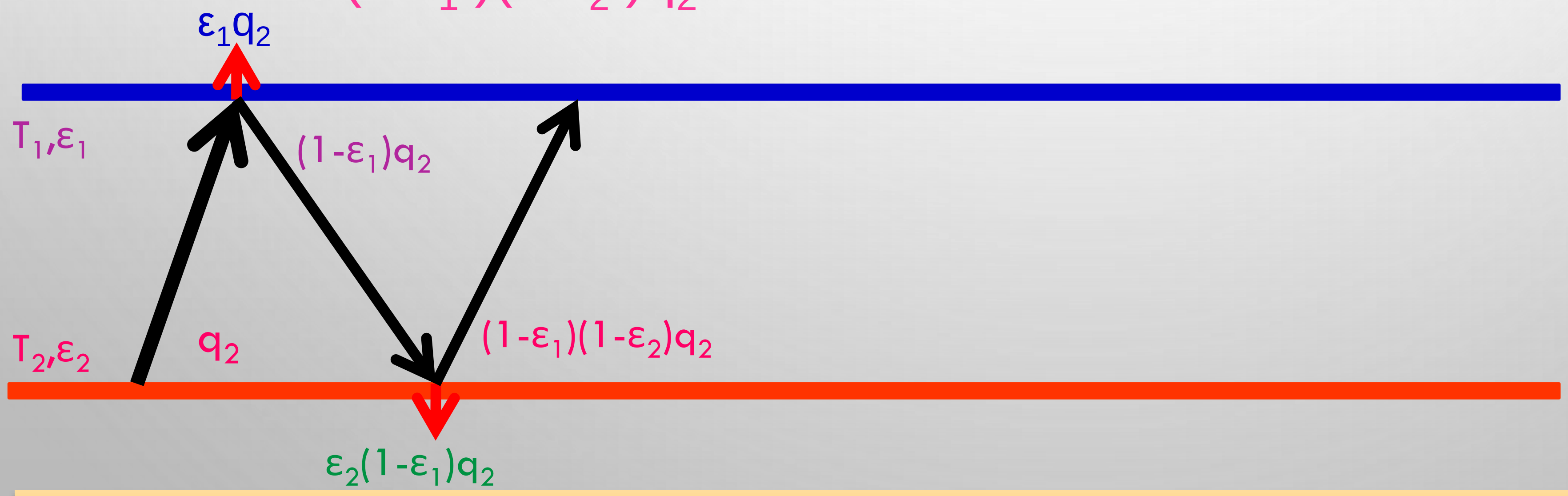
For grey bodies, absoptivity α = emissivity ε

Radiation Heat Exchange Between Two Parallel Plates

Consider radiant flux q_2 emitted by surface 2.

Out of q_2, a fraction $\varepsilon_1 q_2$ will be absorbed by surface 1 and rest $(q_2-\varepsilon_1 q_2)$ will be reflected towards surface 2

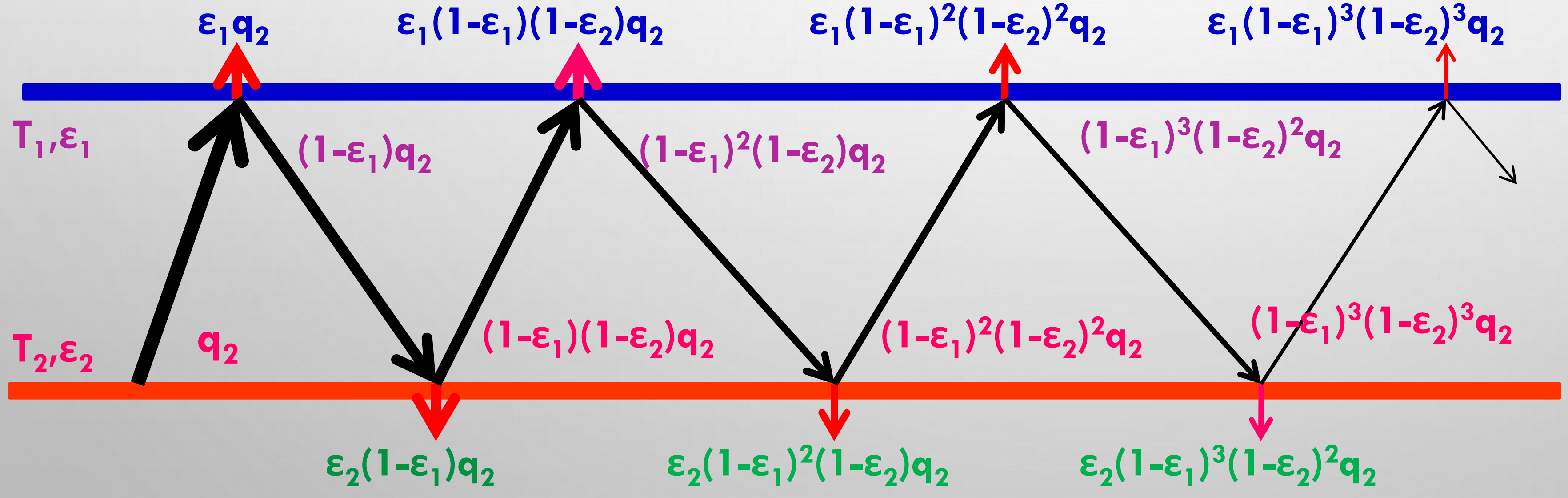
Out of this, $\varepsilon_2(1-\varepsilon_1)q_2$ will be absorbed by surface 2 and balance $(1-\varepsilon_1)(1-\varepsilon_2)q_2$ will be reflected to 1



Radiation Heat Exchange Between Two Parallel Plates

Out of this, $\varepsilon_1(1-\varepsilon_1)(1-\varepsilon_2)q_2$ will be absorbed by surface 1 and balance $(1-\varepsilon_1)^2(1-\varepsilon_2)q_2$ will be reflected back to 2

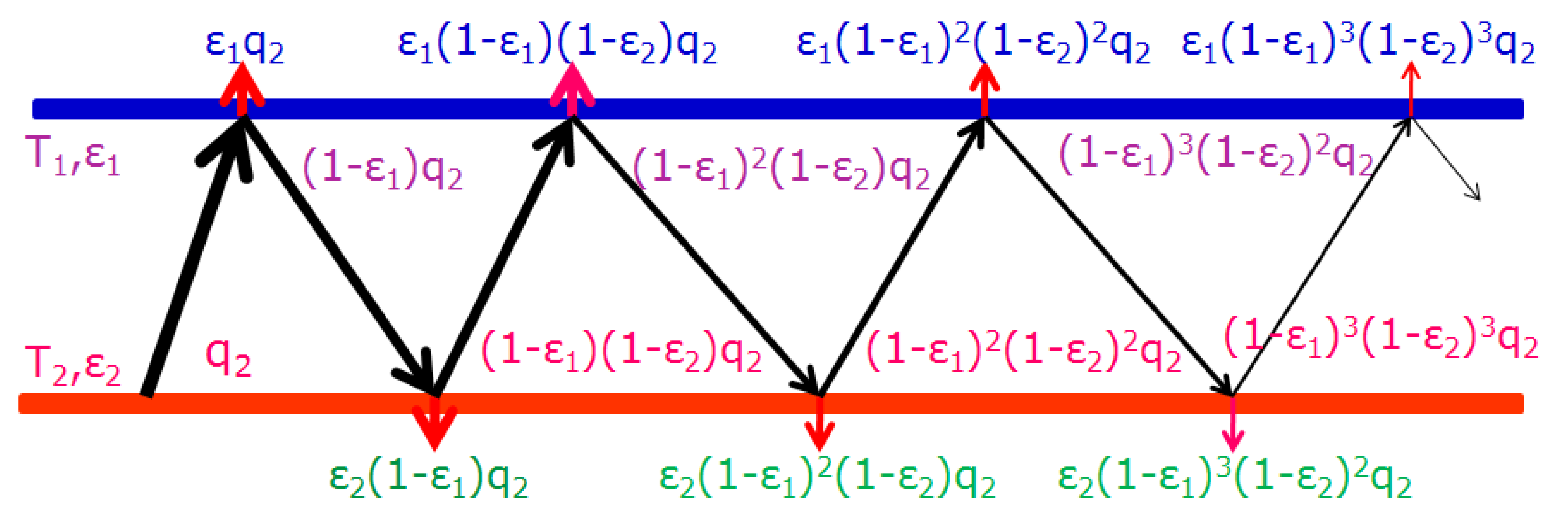
This process of absorption and reflection goes on indefinitely, the quantities involved being successively smaller.



Radiation Heat Exchange Between Two Parallel Plates

Thus, total radiant flux absorbed by surface 1 out of q_2 emitted by surface 2 will be:

$$= \varepsilon_1 q_2 + \varepsilon_1(1-\varepsilon_1)(1-\varepsilon_2)q_2 + \varepsilon_1(1-\varepsilon_1)^2(1-\varepsilon_2)^2 q_2$$
$$+\varepsilon_1(1-\varepsilon_1)^3(1-\varepsilon_2)^3 q_2 + \ldots\ldots\ldots\ldots\ldots\infty$$



Radiation Heat Exchange Between Two Parallel Plates

$$= q_2\varepsilon_1[1 + (1-\varepsilon_1)(1-\varepsilon_2) + (1-\varepsilon_1)^2(1-\varepsilon_2)^2 + (1-\varepsilon_1)^3(1-\varepsilon_2)^3 + \ldots..\infty]$$

$$= \frac{q_2\varepsilon_1}{1-(1-\varepsilon_1)(1-\varepsilon_2)} \qquad = \frac{q_2\varepsilon_1}{1-(1-\varepsilon_1-\varepsilon_2+\varepsilon_1\varepsilon_2)}$$

$$= \frac{q_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Similarly, considering radiation flux q_1 emitted by surface 1 and fraction out of which absorbed by surface 2 can be given as:

$$= \frac{q_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Radiation Heat Exchange Between Two Parallel Plates

Assuming $T_1 > T_2$, net radiant flux absorbed by 2:

$$q_{12} = \frac{q_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2} - \frac{q_2\varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2}$$

$$Since q_1 = \varepsilon_1\sigma {T_1}^4 \& q_2 = \varepsilon_2\sigma {T_2}^4$$

$$q_{12} = \frac{\varepsilon_2\varepsilon_1\sigma {T_1}^4 - \varepsilon_1\varepsilon_2\sigma {T_2}^4}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2}$$

$$q_{12} = \frac{\sigma({T_1}^4 - {T_2}^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

$$Or Q_{12} = \frac{\sigma A({T_1}^4 - {T_2}^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

Shape/Space Resistance:

Heat flow between two black surfaces at temps T_1 & T_2 can be written as:

$$Q_{12} = F_{12}A_1\sigma\left(T_1^{\ 4} - T_2^{\ 4}\right) = \frac{\sigma\left(T_1^{\ 4} - T_2^{\ 4}\right)}{\frac{1}{A_1F_{12}}}$$

$$Here, Equivalent Potential Diff = \sigma\left(T_1^{\ 4} - T_2^{\ 4}\right)$$

$$And\ Equivalent\ RESISTANCE = \frac{1}{A_1F_{12}} = \frac{1}{A_2F_{21}}$$

Due to finite dimensions of the surfaces, 100% of emitted radiation from surface 1 does not fall on surface 2, hence some part of emitted energy go to surroundings, thus lost. This loss is conceptually explained to be caused due to resistance offered by finiteness of dimensions of surfaces and their orientation. Hence, it is called Shape/Space Resistance

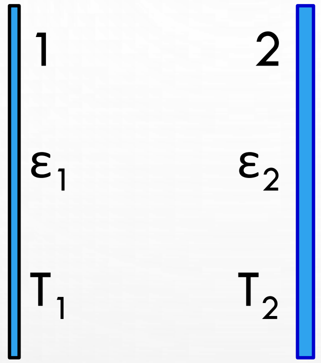
Surface Resistance:

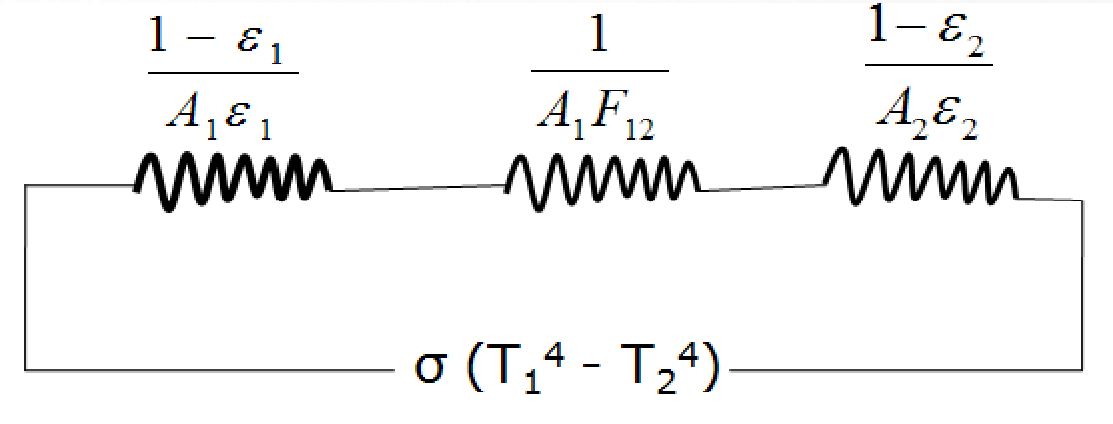
- Black body emits max possible radiation, and its emissivity is taken as 1(the datum). However, Grey bodies emit less due to surface properties; and hence their emissivities are taken as less than 1 (in comparison).
- Therefore, emission of radiation from grey bodies is always less than that of black body. This lesser emission is conceptually assumed to be caused due to a resistance offered by surface of the body as it depends on surface property; the emissivity. This resistance is called Surface Resistance and given as

$$R_{surface} = \frac{1-\varepsilon_1}{A_1\varepsilon_1}(of the surface 1) \& \frac{1-\varepsilon_2}{A_2\varepsilon_2}(of surface 2)$$

Radiation Heat Exchange Between Two Parallel Plates (By Other Method)

$$Q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$



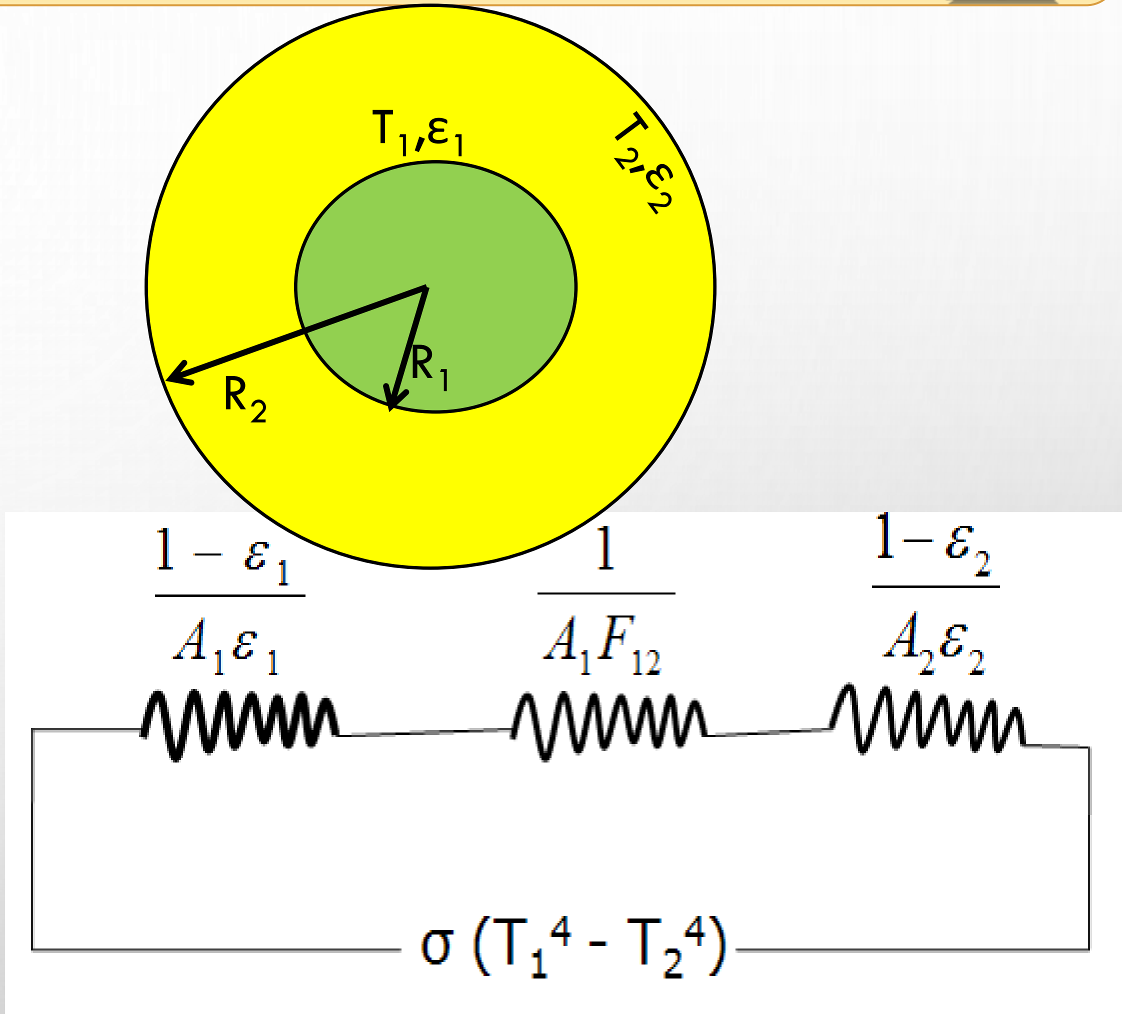


Since $F_{12}=1$ & $A_1=A_2=A$;

$$\frac{Q_{12}}{A} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\varepsilon_1} - 1 + 1 + \frac{1}{\varepsilon_2} - 1}$$

$$\Rightarrow q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

Radiation Heat Exchange Between Two Concentric Infinitely Long Grey Cylinders



$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$F_{12}=1$ as inner cylinder is completely enclosed by 2

Radiation Heat Exchange Between Two Concentric Infinitely Long Grey Cylinders

$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

Putting $F_{12}=1$,
We have:

$$Q_{12} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{A_1}\left[\frac{1}{\varepsilon_1} - 1 + 1 + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]}$$

$$\Rightarrow Q_{12} = \frac{\sigma A_1\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$A_1 = 2\pi R_1 L$

$A_2 = 2\pi R_2 L$

Radiation Heat Exchange Between Two Surfaces

$$\Rightarrow Q_{12} = \frac{\sigma A_1({T_1}^4 - {T_2}^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

This expression is very useful as it can be applied to so many situations:

1. For heat exchange between two concentric spheres; Only diff will be : $A_1=4\pi {R_1}^2$ & $A_2=4\pi {R_2}^2$
2. For eccentric cylinders and spheres
3. For heat exchange between two parallel plates as $A_1 = A_2 = A$
4. For convex/Flat surface completely enclosed by other body as $F_{12}=1$ and $F_{21}=A_1/A_2$

If enclosure (A_2) is very large, $A_1/A_2 \approx 0$; Hence, $Q = \sigma\, \varepsilon_1 A_1({T_1}^4 - {T_2}^4)$

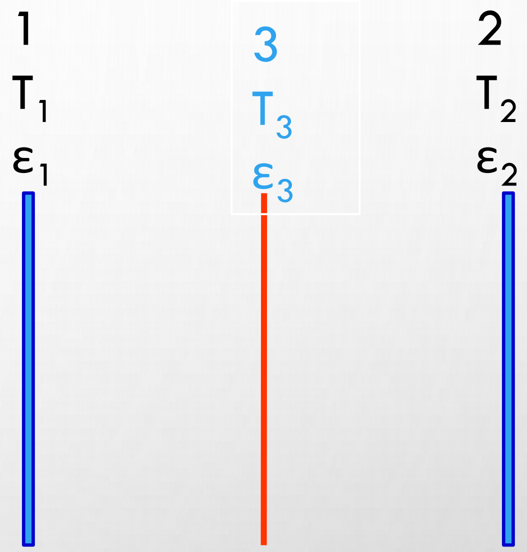
Radiation Shield

- In order to reduce the radiation heat transfer rate between two surfaces, a third surface is inserted between them. This surface is known as Radiation Shield.
- Requirements of Shield (Surface):
  - - Highly reflecting
  - - Lowest emissivity (also absorptivity)
  - - Lowest thickness (thinnest)
- Applications in more effective thermos flasks, for reducing error in temp measurement by thermocouples etc

Radiation Shield

Heat Flow Rate
assuming $T_1 > T_2$:

$$q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$



Now, a shield having both side emissivity ε_3 is placed between the surfaces 1 & 2.

On achieving steady state, the shield will attain steady temp T_3 between T_1 & T_2.

Since T_3 remains steady, that means whatever radiation, the shield is receiving from surface 1, the same it is giving out to surface 2.

Radiation Shield

$$Hence, q_{13} = q_{32} \Rightarrow \frac{\sigma\left({T_1}^4 - {T_3}^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma\left({T_3}^4 - {T_2}^4\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$Substituting \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1 = x and \frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1 = y$$

$$We have, \frac{{T_1}^4 - {T_3}^4}{x} = \frac{{T_3}^4 - {T_2}^4}{y} \Rightarrow \frac{{T_3}^4}{y} + \frac{{T_3}^4}{x} = \frac{{T_1}^4}{x} + \frac{{T_2}^4}{y}$$

$$Or x{T_3}^4 + y{T_3}^4 = y{T_1}^4 + x{T_2}^4 \Rightarrow {T_3}^4 = \frac{y{T_1}^4 + x{T_2}^4}{x + y}$$

Radiation Shield

$Substituting T_3{}^4 in\ q_{13} \exp \rightleftarrows ression;$

$$q_{13} = \frac{\sigma\left[T_1{}^4 - \frac{yT_1{}^4 + xT_2{}^4}{x+y}\right]}{x}$$

$$= \frac{\sigma\left(xT_1{}^4 + yT_1{}^4 - yT_1{}^4 - xT_2{}^4\right)}{x(x+y)}$$

$$\therefore q_{13} = \frac{\sigma.x.\left(T_1{}^4 - T_2{}^4\right)}{x(x+y)} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{x+y}$$

Radiation Shield

$$Substituting x \& y; q_{13} = \frac{\sigma({T_1}^4 - {T_2}^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1\right)}$$

On simplification:

$$q_{13} = \frac{\sigma({T_1}^4 - {T_2}^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{2}{\varepsilon_3} - 1\right)}$$

Since ε_3 will be very small, hence denominator of q_{13} will be very large, therefore, there shall be large reduction of q_{12} to q_{13}.

Radiation Shield

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$;

$$q_{13} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right) + \left(\frac{2}{\varepsilon} - 1\right)} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\left(\frac{2}{\varepsilon} - 1\right) + \left(\frac{2}{\varepsilon} - 1\right)}$$

$$This means, \left(\frac{2}{\varepsilon} - 1\right) is used twice with one shield$$

$$Hence, q_{13} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{(n+1)\left(\frac{2}{\varepsilon} - 1\right)} with n shields$$

Radiation Shield

$$Hence, q_{13} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{(n+1)\left(\frac{2}{\varepsilon} - 1\right)} with n shields$$

$$With ONE shield;\ q_{13} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{2\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{q_{12}}{2}$$

Hence, q_{13} now becomes half of q_{12}

Home Assignment:

$$\Pr \rightleftarrows \rightleftarrows o \rightleftarrows vethat Q_{13} = \frac{\sigma A_1 \left({T_1}^4 - {T_2}^4\right)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right) + \frac{A_1}{A_3}\left(\frac{2}{\varepsilon_3} - 1\right)},$$

when a shield having $\varepsilon_3$emissivity is placed between TWO $\frac{\text{Cylinders}}{\text{Sphheres}}$ 1&2 having emissivities ε_1 & ε_2 ma $\rightleftarrows$ int $\rightleftarrows$ a ined attemps T_1&T_2 having areas A_1& A_2.

Q1: Effective temp of a body having an area of $0.12m^2$ is $527°C$. Calculate the following:

a) Rate of radiation energy emission
b) Intensity of normal radiation
c) Wavelength of max monochromatic emissive power

Solution:

a) Total emission of radiation $Q=\sigma A T^4$

$$Q = 5.67x10^{-8}x0.12x(527+273)^4 = 2786.9W$$

$$b) Intensity of Normal Radiation$$

$$I_n = \frac{q_b}{\pi} = \frac{\sigma T^4}{\pi} = 5.67x10^{-8}x(527+273)^4 = 7392.5W/m^2.sr$$

c) Wavelength of max monochromatic emissive power:

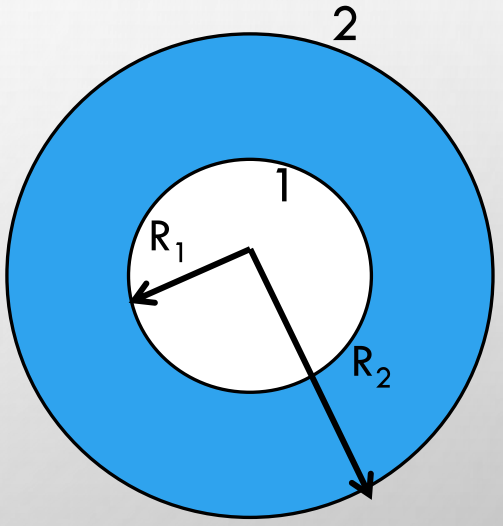
From Wien's Displacement Law;

$$\lambda_m T = 0.0029 mK$$
$$\Rightarrow \lambda_m = \frac{0.0029}{T} = \frac{0.0029}{527 + 273}$$
$$\therefore \lambda_m = 3.625x10^{-6} m = 3.625 \mu m$$

Answer

Q2: A sphere of radius 5cm is concentric with another sphere. Find the radius of the outer sphere so that shape factor of outer sphere wrt inner sphere is 0.6.

Solution:



$A_1=4\pi r_1^2$ $A_2=4\pi r_2^2$ $F_{21}=0.6$

Since sphere 1 is completely enclosed by sphere 2, hence $F_{12}=1$

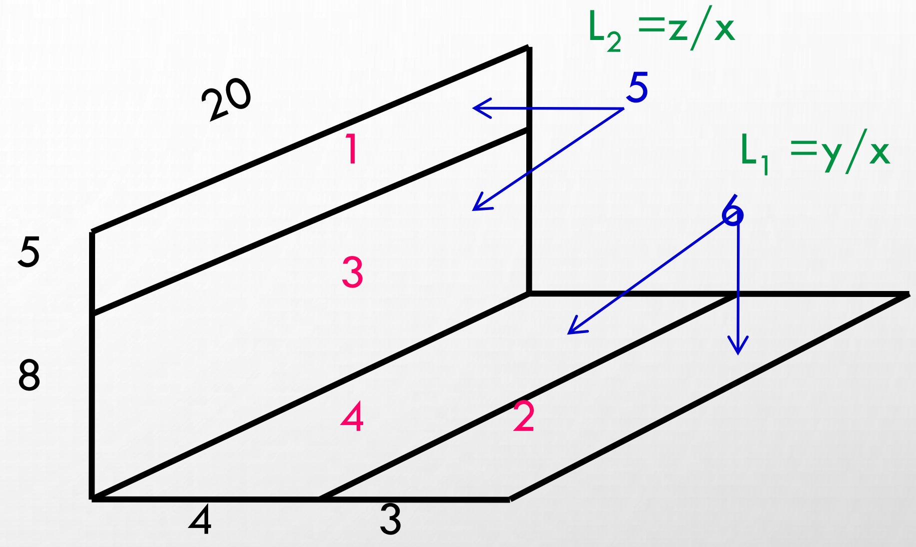
We know that $F_{12}.A_1=F_{21}.A_2$

$Substituti \rightleftarrows ngvalues, wehave;$

$1x4\pi(0.05)^2 = 0.6x4\pi {R_2}^2$

$R_2 = 6.45 cm Answer$

Q3: Find F_{12}.



$$F_{12} = F_{16} - F_{14}$$

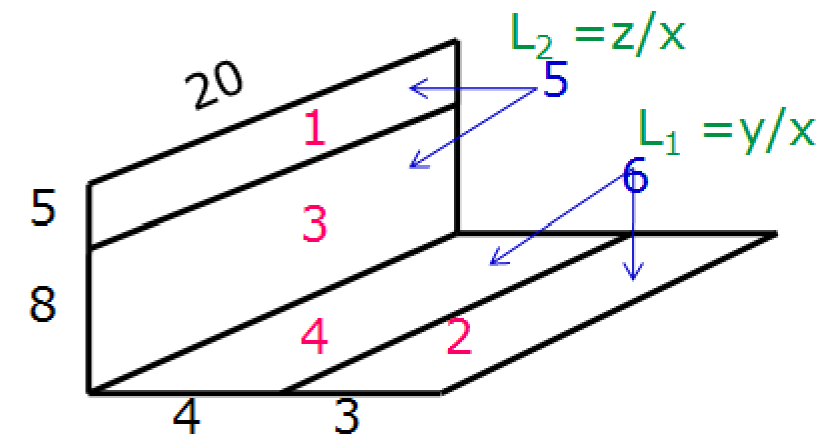
$$= \frac{A_6}{A_1} F_{61} - \frac{A_4}{A_1} F_{41}$$

$$= \frac{A_6}{A_1} (F_{65} - F_{63}) - \frac{A_4}{A_1} (F_{45} - F_{43})$$

$$F_{65}: \frac{L_1}{W} = \frac{7}{20} = 0.35 \& \frac{L_2}{W} = \frac{13}{20} = 0.65$$

From graph: F_{65}=0.32

Solution (Contd):



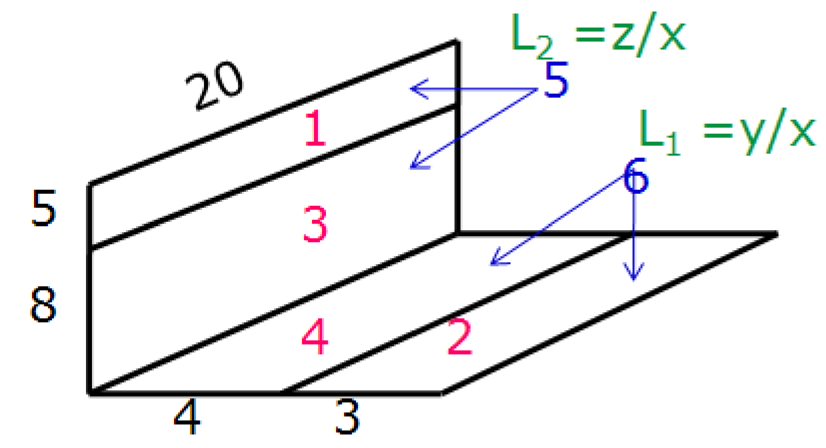
$$F_{63}: \frac{L_1}{W} = \frac{7}{20} = 0.35 \& \frac{L_2}{W} = \frac{8}{20} = 0.4$$

From graph: F_{63}=0.26

$$F_{45}: \frac{L_1}{W} = \frac{4}{20} = 0.2 \& \frac{L_2}{W} = \frac{13}{20} = 0.65$$

From graph: F_{45}=0.36

Solution (Contd):



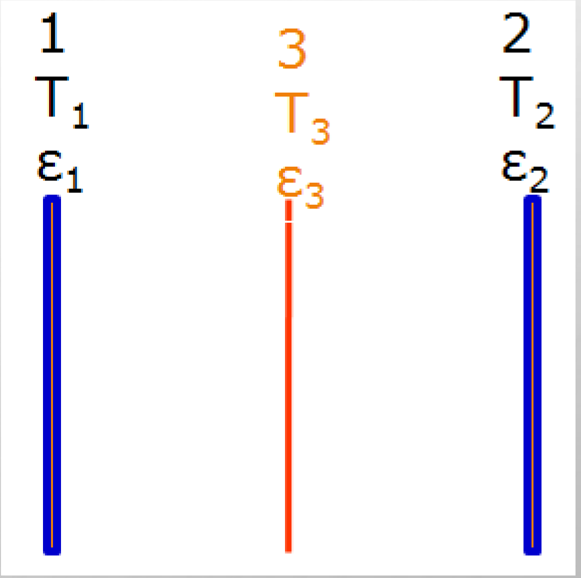
$$F_{43}: \frac{L_1}{W} = \frac{4}{20} = 0.2 \& \frac{L_2}{W} = \frac{8}{20} = 0.4$$

From graph: F_{43}=0.33

$$F_{12} = \frac{A_6}{A_1}(F_{65} - F_{63}) - \frac{A_4}{A_1}(F_{45} - F_{43})$$

$$F_{12} = \frac{7x20}{5x20}(0.32 - 0.26) - \frac{4x20}{5x20}(0.36 - 0.33) = 0.06$$

Q 3: Find out heat transfer rate due to radiation between two infinitely long parallel planes. One plane has emissivity of 0.4 and is maintained at 200°C. Other plane has emissivity of 0.2 and is maintained at 30°C. If a radiation shield (ε=0.5) is introduced between the two planes, find percentage reduction in heat transfer rate and steady state temp of the shield.



Solution:

$$q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$q_{12} = \frac{5.67x10^{-8}[(200+273)^4 - (30+273)^4]}{\frac{1}{0.4} + \frac{1}{0.2} - 1}$$

$$= 363W/m^2$$

Solution (Contd):

$Whenshieldisinserted;$

$$q_{13}=\frac{\sigma\left(T_1{}^4-T_2{}^4\right)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{2}{\varepsilon_3}-1\right)}$$

$$q_{13}=\frac{5.67x10^{-8}[(200+273)^4-(30+273)^4]}{\left(\frac{1}{0.4}+\frac{1}{0.2}-1\right)+\left(\frac{2}{0.5}-1\right)}=248.4W/m^2$$

$$Percentagereduction=\frac{q_{12}-q_{13}}{q_{12}}x100$$

$$=\frac{363-248.4}{363}x100=31.57\%$$

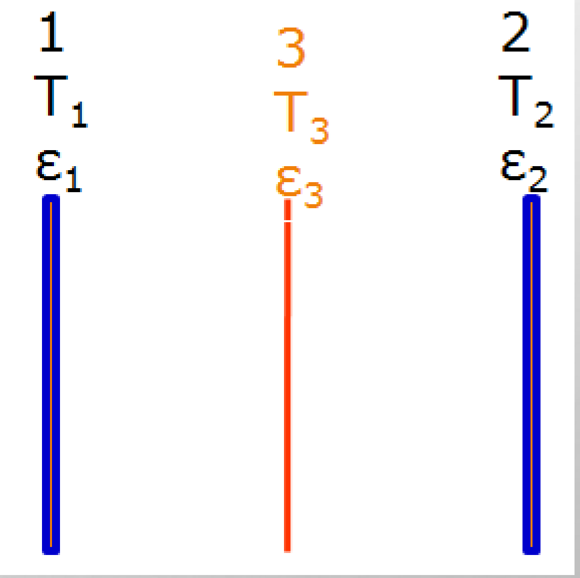
Solution (Contd):

Under Steady State Conditions, we have:

$$q_{13} = q_{32} \Rightarrow \frac{\sigma\left(T_1{}^4 - T_3{}^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma\left(T_3{}^4 - T_2{}^4\right)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$or \frac{\sigma\left[(200+273)^4 - T_3{}^4\right]}{\left(\frac{1}{0.4} + \frac{1}{0.5} - 1\right)} = \frac{\sigma\left[T_3{}^4 - (30+273)^4\right]}{\left(\frac{1}{0.5} + \frac{1}{0.2} - 1\right)}$$

$$\Rightarrow T_3 = 431.67K$$



Gas Radiation

- Gases in many cases are transparent to radiation
- When they absorb and emit radiation, they usually do so only in certain narrow wavelength bands.
- Some gases such as N_2, O_2 and other non-polar gases are essentially transparent to radiation, and they do not emit radiation
- While polar gases like CO_2, H_2O and various hydrocarbon gases absorb and emit radiation to an appreciable extent in narrow wavelength bands.
- For solids and liquids, radiation occurs from thin layer (1μm to 1mm) of surface, hence it is surface phenomenon. However, for gases it is not surface but volumetric phenomenon.

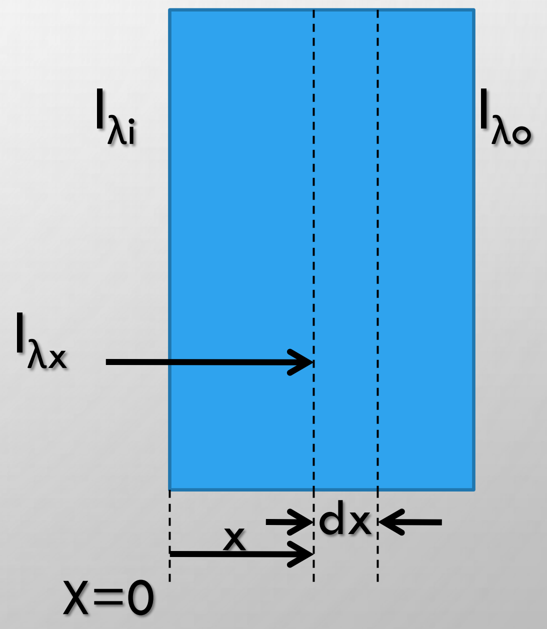
Volumetric Absorption:

- Let a monochromatic beam of radiation having an Intensity $I_{\lambda i}$ impinges on the gas layer of thickness dx as shown in Fig.

- Decrease in intensity resulting from absorption in the layers is proportional to the thickness of layer and intensity of radiation at that point

- Thus;

$$dI_\lambda = -k_\lambda I_\lambda dx;$$
$$where\, k_\lambda\, is\, called\, monochromatic\, absorption\, coefficient$$

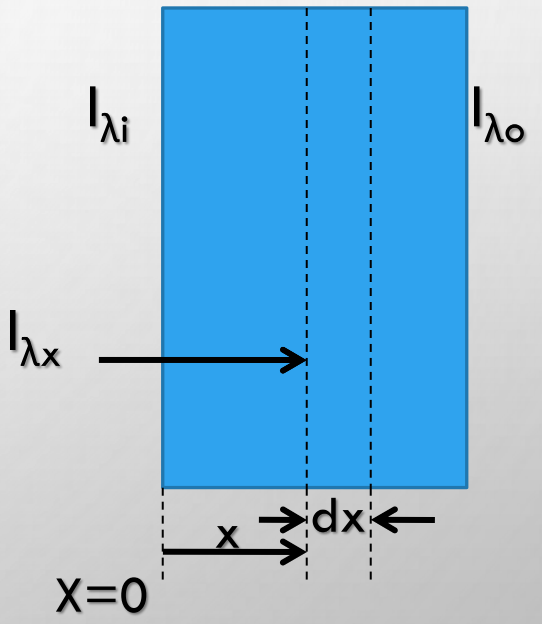


Volumetric Absorption:

- Integrating this equation gives;

$$\int_{I\lambda i}^{I\lambda x} \frac{dI_\lambda}{I_\lambda} = \int_0^x -k_\lambda . dx$$

$$or \frac{I_{\lambda x}}{I_{\lambda i}} = e^{-k_\lambda x}$$



- This is Beer's Law and represents exponential decay of radiation intensity

- we know that monochromatic transmissivity;

$$\tau_\lambda = e^{-k_\lambda x}$$

- If gas is non-reflecting, then;

$$\tau_\lambda + \alpha_\lambda = 1$$
$$and\ hence\ \alpha_\lambda = 1 - \tau_\lambda$$

$$Therefore\ Absorptivity\ \alpha_\lambda = 1 - e^{-k_\lambda . x}$$

$$And, for\ grey\ surface, Emissivity\ \varepsilon_\lambda = \alpha_\lambda = 1 - e^{-k_\lambda . x}$$

Emissivity of CO_2, H_2O Vapor & Gas

- Emissivity of a gas mixture is a function of total pressure (P), partial pressure of a gas(p), gas temperature (T) and characteristic dimension of the system; also known as beam length

$$\therefore \varepsilon = f(P, p, T, L)$$

- When the gas mixture is at 1 atm total pressure, the emissivity of CO2 and H2O vapors are given by following empirical relations;

$$\varepsilon_c = 3.5(p.L)^{0.33}\left[\frac{T}{100}\right]^{3.5}$$

$$\varepsilon_w = 3.5p^{0.8}L^{0.6}\left[\frac{T}{100}\right]^{3}$$

For most practical cases, Mean Beam Length is taken as

$$L = 3.6x\,\frac{Volume\ of\ Gas\ mixture}{Surface\ area\ of\ enclosure}$$

Heat Exchange between Gas Volume & its Enclosure

- Rate of radiant heat transfer from the gas to its enclosure is given by:

$$Q = \varepsilon_g . A_s . \sigma . T_g{}^4 ;$$

$$where \varepsilon_g emissivity of gas emixture$$

$$A_s Enclosure inside surface area$$

$$T_g Gas emixture temp$$

- If the enclosure surface is black, it will absorb all this radiation but it will also emit radiation. Hence net rate, at which the radiation is exchanged between the black enclosure surface at temp T_s and the gas mixture at temp T_g (T_g T_s) is given by:

$$Q = A_s . \sigma [\varepsilon_g . T_g{}^4 - \alpha_g . T_s{}^4]$$

Heat Exchange between Gas Volume & its Enclosure

- If the enclosure surface is grey, the net heat transfer to grey enclosure having emissivity ε_{grey} is given by:

$$\frac{Q_{grey}}{Q_{black}} = \frac{\varepsilon_{grey} + 1}{2}$$